

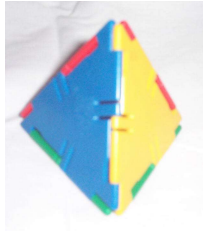
Name \_\_\_\_\_

Group: Blue Yellow

# Summer Educational Enrichment in Math, 2024

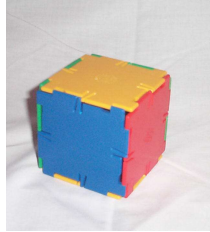
## Math Contest

1. **Platonic Solids:** Name the 5 Platonic Solids and say how many faces they have.  
(Spelling does not matter.)



Name \_\_\_\_\_

Faces \_\_\_\_\_



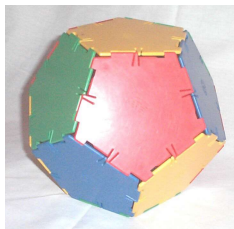
Name \_\_\_\_\_

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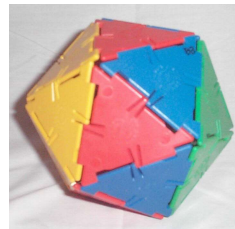
Name \_\_\_\_\_

Faces \_\_\_\_\_



Name \_\_\_\_\_

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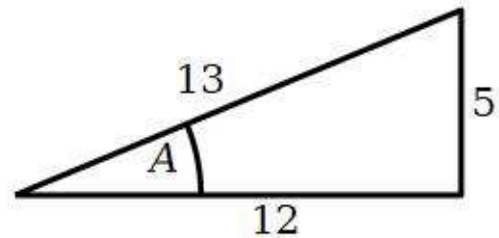
Name \_\_\_\_\_

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2. **Trig Functions:**

For the right triangle at the right, identify the trig functions for the angle  $A$ .

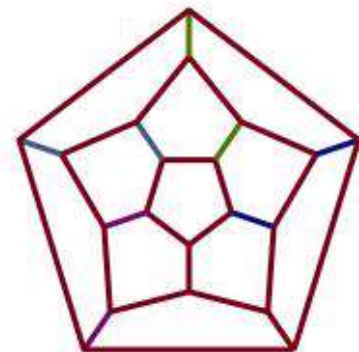
$\sin A = \underline{\hspace{2cm}}$        $\cos A = \underline{\hspace{2cm}}$



3. **Map Coloring:** The map at the right has 11 countries. Color it with as few colors as possible. Countries with a common edge must have different colors. Use the abbreviations:

$R$ =red     $B$ =blue     $G$ =green     $Y$ =yellow     $P$ =purple

Explain why you cannot do it with fewer colors.



4. **Euler numbers:** Consider the octahexahedron made from 6 squares and 8 triangles:



The number of faces is:  $F =$

The number of vertices is:  $V =$   Explain below.

The number of edges is:  $E =$   Explain below.

Explain  $V$ : (Don't use the Euler number.)

Explain  $E$ : (Don't use the Euler number.)

Calculate the Euler number:

Explain how you know the Euler number before counting  $F$ ,  $V$  and  $E$ ?

5. **Balderdice:**

- a. If there are 24 dice remaining at the table, how many total dice at the table would you expect to be 5s and 1s?
- b. If there are 24 dice remaining at the table and you managed to roll a 5 or 1 on all 3 of your own dice, how many total dice at the table would you expect to be 5s and 1s?

6. **Matrices:** Compute the following matrix product:

$$\begin{pmatrix} 2 & 4 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{pmatrix}$$

7. **Birthday Problem:**

a. What is the probability that Polly and Jason have different birthdays (assuming neither was born in a leap year)?

b. If 5 people are in a room, what is the probability that at least 2 of them have birthdays in the same month?

8. **Strings:** You hold 4 strings in your hand. You tie off 2 pairs at the top and 2 pairs at the bottom. When you let go, what is the probability that the strings are all in one loop?

9. **Infinities:** TRUE OR FALSE? There are more natural numbers than there are even numbers.

a. Circle one: TRUE or FALSE

b. Explain your answer.

10. **Hilbert Hotel:** You run the Hilbert Hotel, which has an infinite number of rooms and is full. 2024 guests want rooms for a Math Convention. Explain how you rearrange everyone to accommodate the guests by stating which room the person in room  $N$  needs to move to.

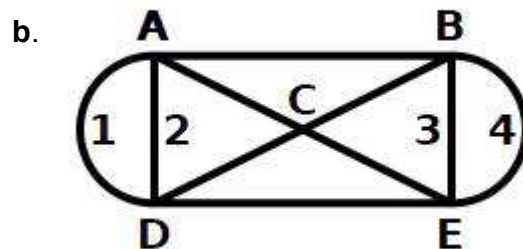
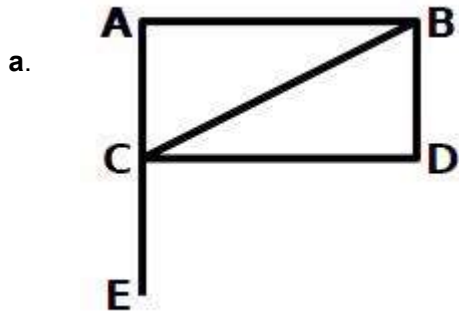
$N \rightarrow$  \_\_\_\_\_

11. **Rational Tangles:** You have two ropes which were tangled using Twists (T) and Rotations (R). The tangle is assigned the rational number  $\frac{-4}{3}$ . Write down the list of Twists and Rotations which will untangle the ropes and the rational number assigned to each intermediate step. (There may be more blanks than you need.)

T/R \_\_\_\_\_  
 #  $\frac{-4}{3} \Rightarrow$  \_\_\_\_\_  $\Rightarrow$  \_\_\_\_\_  $\Rightarrow$  \_\_\_\_\_  $\Rightarrow$  \_\_\_\_\_  $\Rightarrow$  \_\_\_\_\_  $\Rightarrow$  \_\_\_\_\_  $\Rightarrow$  \_\_\_\_\_

T/R \_\_\_\_\_  
 #  $\Rightarrow$  \_\_\_\_\_  $\Rightarrow$  \_\_\_\_\_  $\Rightarrow$  \_\_\_\_\_  $\Rightarrow$  \_\_\_\_\_  $\Rightarrow$  \_\_\_\_\_  $\Rightarrow$  \_\_\_\_\_  $\Rightarrow$  \_\_\_\_\_

12. **Euler Paths:** For each graph below determine whether the graph is traversable (in other words, determine whether you can trace each edge of it exactly once without lifting a pencil). If it is not traversable, give a reason for your answer. If it is traversable, show an Euler path (*Start  $\neq$  Finish*) or an Euler circuit (*Start = Finish*) using arrows and numbers along edges.



13. **Cryptography:**

- a. Decode the following message which was encoded using a shift cipher:  
 IZIVC WUYEVI MW E VIGXERKPI.

- b. Decode the following message, which has 45 letters:  
 IESOH DGANE ORNOM NEDTS OEHLA  
 TNAIM LEMKI IGIEA KGDTM

14. **Pop-Tac-Toe:** It is Blue's turn.

Can Blue win on this turn?

Circle one:

Yes          No

If so, which square should Blue play on?

If there is more than one answer,  
just list one winning play.

Play \_\_\_\_\_



A8	B8	C8	D8	E8	F8	G8	H8
A7	B7	C7	D7		F7	G7	H7
A6	B6	C6	D6	E6	F6	G6	H6
	B5		D5		F5		H5
A4	B4	C4	D4	E4		G4	H4
A3	B3		D3	E3	F3	G3	H3
A2	B2	C2	D2	E2	F2	G2	H2
A1	B1	C1	D1		F1	G1	H1



15. **Kenken:** Solve the Kenken:

<b>1-</b>	<b>2-</b>		<b>10x</b>	
	<b>9+</b>	<b>9+</b>		
<b>4</b>		<b>2-</b>		
<b>6+</b>		<b>2÷</b>		<b>10+</b>
	<b>2÷</b>			

16. Solve the cryptogram:

DKCZW VM UDT KIT, DKCZW VM CGHD,

SKJDH K SKI ODKZVOW, FDKZVOW KIT FGHD.

– UDIXKSGI QCKIJZGI

HINT: D → E