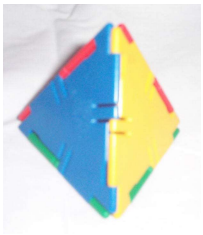
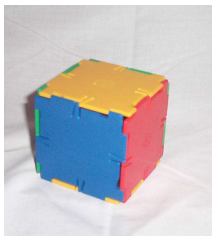
Summer Educational Enrichment in Math
2025 Math Contest - Solutions

1. **Platonic Solids**: Name the 5 Platonic Solids and say how many faces they have. (Spelling does not matter.)



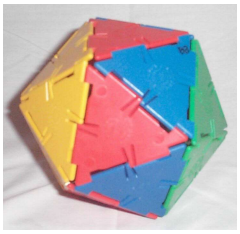
Name Tetrahedron
Faces 4



Name Hexahedron or Cube
Faces 6

Name Octahedron
Faces 8

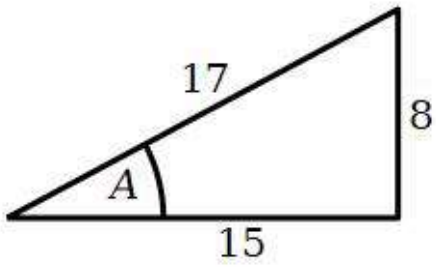
Name Dodecahedron
Faces 12



Name Icosahedron
Faces 20

2. **Trig Functions**:

For the right triangle at the right, identify the trig functions for the angle A.



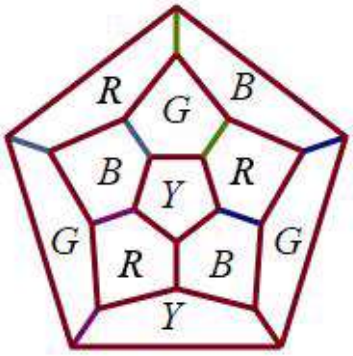
$\sin A =$_____ $\cos A =$_____

Solution:

$$\sin A = \frac{Opp}{Hyp} = \frac{8}{17} \qquad \cos A = \frac{Adj}{Hyp} = \frac{15}{17}$$

3. **Map Coloring**: The map at the right has 11 countries.
Color it with as few colors as possible.
Countries with a common edge must have different colors.
Use the abbreviations:
R=red B=blue G=green Y=yellow P=purple

Explain why you cannot do it with fewer colors.



Solution: It can be done with 4 colors. This is a posible solution.
It can't be done with 3 colors because the central yellow country is surrounded by 5 countries which can start alternating between 2 colors, blue and red, but this cannot go all the way around because 5 is odd. So there must be a 4th color, green.

4. **Euler numbers**: Consider the octahexahedron made from 6 squares and 8 triangles:

The number of faces is: $\boxed{F = 14}$

The number of vertices is: $\boxed{V = 12}$ Explain below.

The number of edges is: $\boxed{E = 24}$ Explain below.

Explain V: **Solution**: The 6 squares have 4 vertices and the 8 triangles have 3 vertices for a total of $6 \times 4 + 8 \times 3 = 48$ vertices, counting each vertex for each face, but each vertex belongs to 4 faces. So we divide to get $48/4 = 12$ vertices.

Explain E: **Solution**: The 6 squares have 4 edges and the 8 triangles have 3 edges for a total of $6 \times 4 + 8 \times 3 = 48$ edges, counting each edge for each face, but each edge belongs to 2 faces. So we divide to get $48/2 = 24$ edges.

Calculate the Euler number: **Solution**: $F + V - E = 14 + 12 - 24 = 2$

Explain how you know the Euler number before counting F, V and E?

Solution: There are no holes.

5. **Balderdice**:

a. If there are 24 dice remaining at the table, how many total dice at the table would you expect to be 5s and 1s?

Solution: In 24 random dice you would expect 4 of each of the 6 numbers. So 4 of 5s and 4 of 1s. So a total of 8.

b. If there are 24 dice remaining at the table and you managed to roll a 5 or 1 on all 3 of your own dice, how many total dice at the table would you expect to be 5s and 1s?

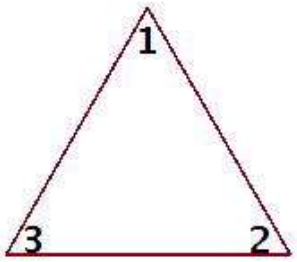
Solution: You have 3 dice. Of the 21 remaining dice you would expect $\frac{1}{3}$ of them to be 5s and 1s, or 7. With your 3, that's a total of 10.

6. **Matrices**: Compute the following matrix product:

$$\begin{pmatrix} 2 & 4 \\ 5 & 3 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} \_\_\_ \\ \_\_\_ \end{pmatrix}$$

Solution: $\begin{pmatrix} 2 & 4 \\ 5 & 3 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \cdot 3 + 4 \cdot 2 \\ 5 \cdot 3 + 3 \cdot 2 \end{pmatrix} = \begin{pmatrix} 14 \\ 21 \end{pmatrix}$

7. **Planar Groups**: Let L, R and N be the standard rotations on the triangle at the right (Left rotate, Right rotate, and Nothing)



a. Complete the Multiplication Table if $\otimes$ means "first apply the rotation at the left of the table, then apply the rotation at the top of the table":

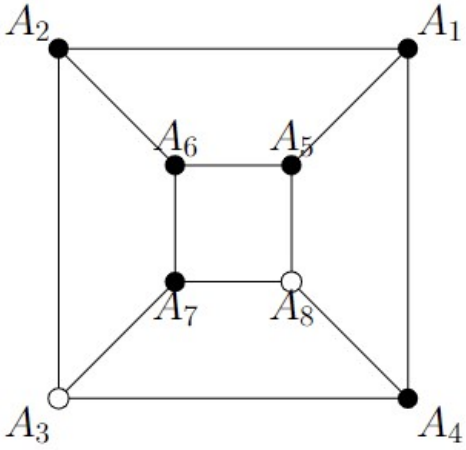
Solution:

$\otimes$	R	L	N
R	L	N	R
L	N	R	L
N	R	L	N

b. List the names of the four properties that make this a group. You do not have to show these properties are true.

Solution: Closed, Associative, Identity, and Inverses

8. **Lights Out**: In the puzzle at the right, the black lights are off and the white lights are on. Which lights should you press to get them all to be off?



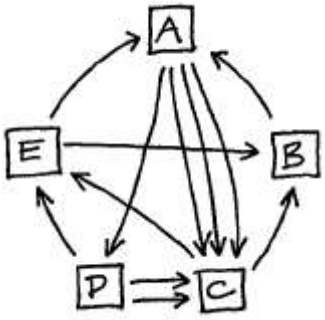
Solution: Press A_2 and A_5.
Then A_1 and A_6 will flip twice, leaving them off.
And A_3 and A_8 will flip once, turning them off.

9. **Sum Fun Puzzle**:
In the square at the right, circle the numbers so that the circled numbers in each row add up to the number at the left and those in each column add up to the number at the top.

	18	6	4	8
7	3	7	4	8
12	5	5	7	7
9	8	1	1	8
8	7	9	3	1

Solution:
Row 1: 3 & 4
Row 2: second 5 & second 7
Row 3: first 8 & first 1
Row 4: 7 & 1

10. **Search** & **Rank**: The graph below represents a model web with five webpages. Fill in the entries of the corresponding probability matrix M. For example, the entry which lies in row B and column C is the probability of linking from C to B which is $1/2$.



$$M = \begin{array}{c} \begin{matrix} A & B & C & D & E \end{matrix} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{3}{4} & 0 & 0 & \frac{2}{3} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \end{pmatrix} \end{array} \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix}$$

By examining the graph, try to determine which webpage will be visited most frequently on a random walk through the web. Answer: $\boxed{A}$

11. **Infinities**: TRUE OR FALSE?

There are more natural numbers than there are even numbers.

a. Circle one: TRUE or $\boxed{\text{FALSE}}$

b. Explain your answer.

Solution: There is a one-to -one correspondence between the natural numbers and the even numbers. Each natural number is paired with its double.

$$\begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & \cdots \\ 2 & 4 & 6 & 8 & 10 & 12 & \cdots \end{matrix}$$

So these sets have the same cardinality, $\aleph_0$.

12. **Hilbert Hotel**: You run the Hilbert Hotel, with an infinite number of rooms which is full. 2025 new guests want rooms for a Math Convention. Explain how you rearrange everyone to accommodate the guests by stating which room the person in room N needs to move to.

Solution: $N \rightarrow \boxed{N + 2025}$

13. **Permutations**: Let $\otimes$ be defined as "apply the first permutation, then apply the second permutation". On the set $\{1,2,3,4,5,6\}$, compute each of the following:

a. $(1243)(56) \otimes (134562) =$ $\boxed{(25)}$

b. The inverse of (134562) is $\boxed{(126543)}$

14. **Catalan Numbers**:

a. How many different ways are there to subdivide a convex hexagon into triangles using non-crossing diagonals? You don't need to list the subdivisions, just their total number.

Solution: $N =$ ____14____

b. How many strings of 6 parentheses are there, so that each of 3 opening parentheses is matched by a later closing one? For example, for 4 parentheses there are only two choices, (()) and ()().You don't need to list all the strings, just their total number.

Solution: $N =$ ____5____Specifically: ((())), (()()), (())(), ()(()), ()()()

15. **Manufacturing Matrices**: You have decided you want to use up all of your sugar and flour before your big trip by baking coffee cake and brownies for your friends. Each coffee cake requires 7 ounces of flour and 10 ounces of sugar; each brownie requires 2 ounces of flour and 3 ounces of sugar. You have 34 ounces of flour and 50 ounces of sugar on hand. Set up an equation with matrices to describe how many coffee cakes and how many brownies should you make to use up all of the flour and sugar.

Extra Credit: Solve it.

Solution: $\begin{aligned} 7C + 2B &= 34 \\ 10C + 3B &= 50 \end{aligned}$ or $\begin{pmatrix} 7 & 2 \\ 10 & 3 \end{pmatrix}\begin{pmatrix} C \\ B \end{pmatrix} = \begin{pmatrix} 34 \\ 50 \end{pmatrix}$

Extra Credit: The inverse matrix is $\begin{pmatrix} 3 & -2 \\ -10 & 7 \end{pmatrix}$. So:

$$\begin{pmatrix} C \\ B \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ -10 & 7 \end{pmatrix}\begin{pmatrix} 34 \\ 50 \end{pmatrix} = \begin{pmatrix} 3 \times 34 - 2 \times 50 \\ -10 \times 34 + 7 \times 50 \end{pmatrix} = \begin{pmatrix} 2 \\ 10 \end{pmatrix}$$

16. **Manufacturing Errors**: Note that $0.1 + 0.0045 = 0.1045$ and $0.1 + 0.0055 = 0.1055$.
In Maple, you type in the following three commands. Fill in the outputs:

> Digits:=3:

> 0.1 + 0.0045;

_____

> 0.1 + 0.0055;

_____

Solution:

> 0.1 + 0.0045;

_____0.104_____

> 0.1 + 0.0055;

_____0.106_____

17. **Rubik's cube**: Let M be the sequence of Rubik's cube moves $M = FLD^{-1}LLF^{-1}LR^{-1}DR$. This sequence of moves only switches two corner cubies in the top row, but leaves the bottom 2 rows a jumbled mess.

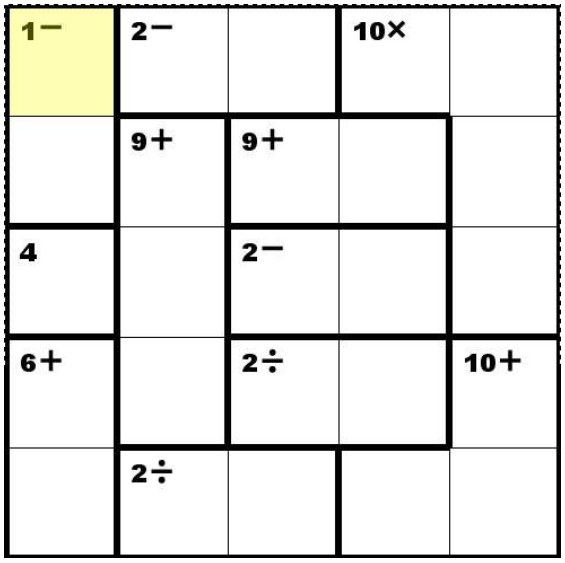
a. What is M^{-1}?

Solution: $M^{-1} = R^{-1}D^{-1}RL^{-1}FL^{-1}L^{-1}DL^{-1}F^{-1}$

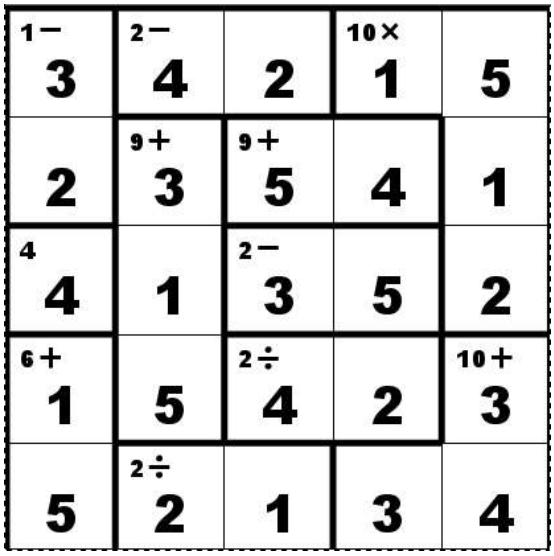
b. Explain how the sequence of moves M can be used as a strategy toward solving a Rubik's Cube.

Solution: Apply M to switch two corner cubies, then rotate the top row and apply M^{-1}. The bottom 2 rows will be fixed and only 3 corner cubies will be switched.

18. **Kenken**: Solve the Kenken:



Solution:



19. Solve the cryptogram:

S XDZHVT RWV TDEDZ OSID S OAHCSJD,

TDEDZ CZADI STNCWATM TDR.

– SFGDZC DATHCDAT

HINT: D → E

A PERSON WHO NEVER MADE A MISTAKE, NEVER TRIED ANYTHING NEW.
– ALBERT EINSTEIN