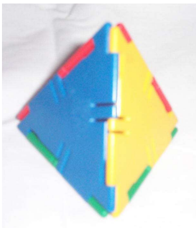
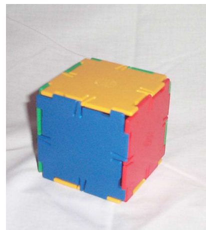
Summer Educational Enrichment in Math
2026 Math Contest - Solutions

1. **Platonic Solids**: Name the 5 Platonic Solids and say how many faces they have. (Spelling does not matter.)

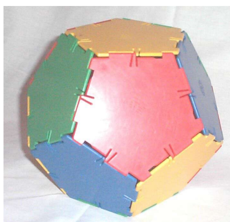


Name Tetrahedron
Faces 4



Name Hexahedron or Cube
Faces 6

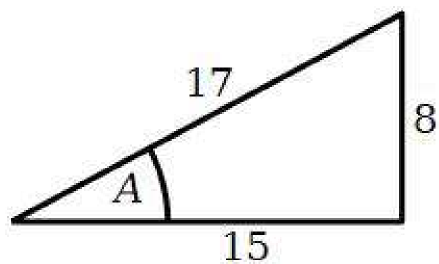
Name Octahedron
Faces 8



Name Dodecahedron
Faces 12

Name Icosahedron
Faces 20

2. **Trig Functions**:
For the right triangle at the right, identify the trig functions for the angle A.



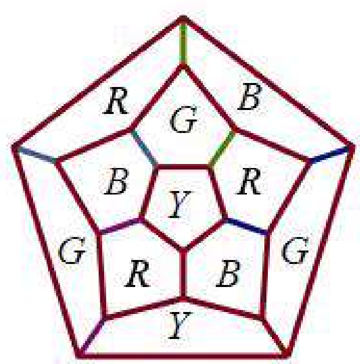
$\sin A =$______ $\cos A =$______

Solution:

$$\sin A = \frac{Opp}{Hyp} = \frac{8}{17} \qquad \cos A = \frac{Adj}{Hyp} = \frac{15}{17}$$

3. **Map Coloring**: The map at the right has 11 countries.
Color it with as few colors as possible.
Countries with a common edge must have different colors.
Use the abbreviations:
R=red B=blue G=green Y=yellow P=purple

Explain why you cannot do it with fewer colors.



Solution: It can be done with 4 colors. This is a posible solution.
It can't be done with 3 colors because the central yellow country is surrounded by 5 countries which can start alternating between 2 other colors, blue and red, but this cannot go all the way around because 5 is odd. So there must be a 4^{th} color, green.

4. **Euler numbers**: Consider the octahexahedron made from 6 squares and 8 triangles:

The number of faces is: $\boxed{F = 14}$

The number of vertices is: $\boxed{V = 12}$ Explain below.

The number of edges is: $\boxed{E = 24}$ Explain below.

Explain V: **Solution**: The 6 squares have 4 vertices and the 8 triangles have 3 vertices for a total of $6 \times 4 + 8 \times 3 = 48$ vertices, counting each vertex for each face, but each vertex belongs to 4 faces. So we divide to get $48/4 = 12$ vertices.

Explain E: **Solution**: The 6 squares have 4 edges and the 8 triangles have 3 edges for a total of $6 \times 4 + 8 \times 3 = 48$ edges, counting each edge for each face, but each edge belongs to 2 faces. So we divide to get $48/2 = 24$ edges.

Calculate the Euler number: **Solution**: $F + V - E = 14 + 12 - 24 = 2$

Explain how you know the Euler number before counting F, V and E?

Solution: There are no holes.

5. **Balderdice**:

a. If there are 24 dice remaining at the table, how many total dice at the table would you expect to be 5s and 1s?

Solution: In 24 random dice you would expect 4 of each of the 6 numbers.
So 4 of 5s and 4 of 1s. So a total of 8.

b. If there are 24 dice remaining at the table and you managed to roll a 5 or 1 on all 3 of your own dice, how many total dice at the table would you expect to be 5s and 1s?

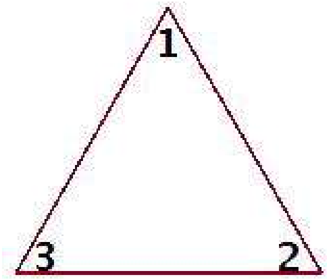
Solution: You have 3 dice. Of the 21 remaining dice you would expect $\frac{1}{3}$ of them to be 5s and 1s, or 7. With your 3, that's a total of 10.

6. **Matrices**: Compute the following matrix product:

$$\begin{pmatrix} 2 & 4 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} \underline{\quad\quad} \\ \underline{\quad\quad} \end{pmatrix}$$

Solution: $$\begin{pmatrix} 2 & 4 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \cdot 3 + 4 \cdot 2 \\ 5 \cdot 3 + 3 \cdot 2 \end{pmatrix} = \begin{pmatrix} \underline{\ 14\ } \\ \underline{\ 21\ } \end{pmatrix}$$

7. **Planar Groups**: Let L, R and N be the standard rotations on the triangle at the right (Left rotate, Right rotate, and Nothing)



a. Complete the Multiplication Table if $\otimes$ means "first apply the rotation at the left of the table, then apply the rotation at the top of the table":

Solution:

$\otimes$	R	L	N
R	L	N	R
L	N	R	L
N	R	L	N

b. List the names of the four properties that make this a group. You do not have to show these properties are true.

Solution: Closed, Associative, Identity, and Inverses

8. **Manufacturing Matrices**: You have decided you want to use up all of your sugar and flour before your big trip by baking cookies and brownies for your friends. Each cookies requires 7 ounces of flour and 10 ounces of sugar; each brownie requires 2 ounces of flour and 3 ounces of sugar. You have 34 ounces of flour and 50 ounces of sugar on hand. Set up an equation with matrices to describe how many cookies and how many brownies should you make to use up all of the flour and sugar.
Extra Credit: Solve it.

Solution: $\begin{aligned} 7C + 2B &= 34 \\ 10C + 3B &= 50 \end{aligned}$ or $\begin{pmatrix} 7 & 2 \\ 10 & 3 \end{pmatrix}\begin{pmatrix} C \\ B \end{pmatrix} = \begin{pmatrix} 34 \\ 50 \end{pmatrix}$

Extra Credit: The inverse matrix is $\begin{pmatrix} 3 & -2 \\ -10 & 7 \end{pmatrix}$. So:

$$\begin{pmatrix} C \\ B \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ -10 & 7 \end{pmatrix}\begin{pmatrix} 34 \\ 50 \end{pmatrix} = \begin{pmatrix} 3 \times 34 - 2 \times 50 \\ -10 \times 34 + 7 \times 50 \end{pmatrix} = \begin{pmatrix} 2 \\ 10 \end{pmatrix}$$

9. **Diophantine Equations**: Find the Greatest Common Divisor of 315 and 84 using the Euclidean Algorithm.
Extra Credit: Find an integer solution to $315x + 84y = 21$.

Solution: The last remainder in the Euclidean Algorithm is the GCD.

$$\begin{aligned} \underline{315} &= \underline{84}(3) + \underline{63} \\ \underline{84} &= \underline{63}(1) + \underline{\mathbf{21}} \\ \underline{63} &= \underline{21}(3) + 0 \end{aligned}$$

The Greatest Common Divisor is 21.

Extra Credit: $63 = 315 - 84(3)$. So

$21 = 84 - 63(1) = 84 - [315 - 84(3)](1) = 315(-1) + 84(1 + 3) = 315(-1) + 84(4)$

So $x = -1$ and $y = 4$.

10. Birthday Problem:

a. What is the probability that Polly and Jason have different birthdays (assuming neither was born in a leap year)?

Solution: $1 \cdot \frac{364}{365} = \frac{364}{365}$

b. If 5 people are in a room, what is the probability that at least 2 of them have birthdays in the same month?

Solution: The probability they all have birthdays in different months.is

$$p = \frac{11}{12} \times \frac{10}{12} \times \frac{9}{12} \times \frac{8}{12} = \frac{11}{12} \times \frac{5}{6} \times \frac{3}{4} \times \frac{2}{3} = \frac{55}{144}$$

So the probability they are not all different is $1 - p = 1 - \frac{55}{144} = \frac{89}{144}$.

11. Strings: You hold 4 strings in your hand. You tie off 2 pairs at the top and 2 pairs at the bottom. When you let go, what is the probability that the strings are all in one loop?

Solution: The top pairings don't matter. When you connect 1 bottom string to 1 of the 3 others, 2 will allow for a single loop. So the probability is $\frac{2}{3}$. The remaining 2 strings are then tied. So the overall probability is $\frac{2}{3}$.

12. Sum Fun Puzzle:

In the square at the right, circle some numbers so that the circled numbers in each row add up to the number at the left and those in each column add up to the number at the top.

	18	6	4	8
7	3	7	4	8
12	5	5	7	7
9	8	1	1	8
8	7	9	3	1

Solution:

Row 1: 3 & 4

Row 2: second 5 & second 7

Row 3: first 8 & first 1

Row 4: 7 & 1

13. Knights Out: Assume we are using the the standard "In, Out, In, Out..." method of elimination.

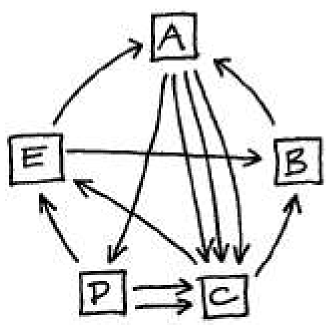
  a. If there are 27 knights around the round table, which knight survives?

  Solution: $2(27-16)+1 = 23$

  b. If there are K knights around the round table, which knight survives?

  Solution: $2(K-2^N)+1$ where 2^N is the largest power of 2 which is less than K.

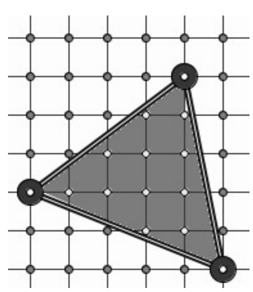
14. **Search** & **Rank**: The graph below represents a model web with five webpages. Fill in the entries of the corresponding probability matrix M. For example, the entry which lies in row B and column C is the probability of linking from C to B which is $1/2$.



$$M = \begin{array}{cc} \begin{matrix} \quad A & \quad B & \quad C & \quad D & \quad E \end{matrix} & \\ \begin{pmatrix} 0 & 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{3}{4} & 0 & 0 & \frac{2}{3} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \end{pmatrix} & \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} \end{array}$$

By examining the graph, try to determine which webpage will be visited most frequently on a random walk through the web. Answer: $\boxed{A}$

15. **Pick's Theorem**: For the shaded region below, state the number of internal points and the number of boundary points, and find the area using Pick's Theorem:



$\boxed{I = 11}$ $\boxed{B = 3}$ $\boxed{A = 11 + \frac{3}{2} - 1 = 11.5}$

16. **Pigeonhole Principle**: There are 37 students at SEE-Math this year. Is it true that at least 4 of you are born in the same month? Explain why or why not.

Solution: Yes! There are 12 months. To avoid having 4 students in any month, you first assign 3 students to each month. That accounts for 36 students. The remaining 1 student must be assigned some month. But then at least one month has 4 students.

17. **Penrose Tilings**: What is one way to check if a tiling is aperiodic?

Solution: 1) A tiling is aperiodic, if it has 5-fold rotational symmetry.
2) A tiling is aperiodic, if it has an irrational ratio between the number of two different tile-types (but does not have exactly one axis of translational symmetry.)

18. **Kenken**: Solve the Kenken:

1−	2−		10×	
	9+	9+		
4		2−		
6+		2÷		10+
	2÷			

Solution:

1− 3	2− 4	2	10× 1	5
2	9+ 3	9+ 5	4	1
4 4	1	2− 3	5	2
6+ 1	5	2÷ 4	2	10+ 3
5	2÷ 2	1	3	4

19. Solve the cryptogram:

H S R S X A S V V C E F S Y X C S Y V,

H M J J M G Y P X M I W M R Q E X L I Q E X M G W.

M G E R E W W Y V I C S Y Q M R I E V I

W X M P P K V I E X I V.

– E P F I V X I M R W X I M R

HINT: E → A

DO NOT WORRY ABOUT YOUR DIFFICULTIES IN MATHEMATICS. I CAN ASSURE YOU MINE ARE STILL GREATER. – ALBERT EINSTEIN